A varifold perspective on discrete surfaces

Blanche Buet

Joint work with : Gian Paolo Leonardi (Trento), Simon Masnou (Lyon) and Martin Rumpf (Bonn).

Abstract : We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well.

While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We prove nice convergence properties involving a natural growth assumption : the scale of regularization must be large with respect to the accuracy of the discretization. We performed numerical computations of mean curvature and Gaussian curvature on point clouds in $\mathbb{R}^3$ to illustrate this approach. Building on the explicit expression of approximate mean curvature we propose, we perform mean curvature flow of point cloud varifolds beyond the formation of singularities and we recover well-known soap films.